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TDOA based Geolocation using IRLS Algorithm

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Abstract: The geolocation method using the wireless signal processing system is widely used in many industrial areas. The time difference of arrival (TDOA) method is one of the most commonly used geolocation methods. The TDOA signal based geolocation system can estimate the position of a mobile object by using at least three base stations in two dimensional spaces. In geolocation problem, the precise estimation of mobile's position is the most significant issue. The measurement noise that is contained in measured TDOA data causes the estimation inaccuracy in a mobile geolocation. In this paper, the objective function that represents the scalar error of position estimation is formulated using the concept of L_p-norm approximation. Also, we suggest the iterative reweighted least square (IRLS) scheme for minimizing of the objective function. The optimal solution can be obtained using the limited measurement data through the reweighted iteration process of the IRLS scheme.

Keywords: Geolocation; Time difference of arrival; Iterated reweighted least square; Obtimization method

I. INTRODUCTION

In the past few years, the geolocation problem has received great interests in the mobile position based industry such as wireless communication, target tracking, augemented reality platform, guidance, and many types of mobile application. Although the global positioning system (GPS) is the most widely used geolocation method, there is a limitation of the measurement accuracy due to jamming. Recently, the active radio signal based geolocation methods such as time of arrival (TOA), angle of arrival (AOA), and received signal strength (RSS) have been actively researched and utilized for replacement of GPS in specific environment such as downtown and indoor position [1]. TDOA method is one of the widely used geolocation techniques as well. TDOA signal based geolocation derives the mobile position using the arrival time difference of the signal from a mobile at each base station [2]. The geolocation method using TDOA measurements has a strength in that the time synchronization among base stations is unnecessary since the cross correlation method among all base stations can measure the TDOA data [3]. However, the geolocation using radio signal propagation also needs to overcome an accuracy problem due to the measurement noise caused by the signal interference and disruption [4], [5].

In the geolocation problem, the measurement noise is the major factor which causes the estimation inaccuracy of position estimation. For improvement of the geolocation estimation accuracy, Wei [6] applied multidimensional scaling (MDS) analysis based estimator to the passive moving target localization using time and frequency difference of arrival measurements. MDS based estimator was mathematically simple and can be easily implemented in a real time system. Blandin [7] suggested several multi-source TDOA estimation methods based on angular spectra and clustering inspired from signal to noise ratio wheighting. In order to find an approximate global solution to geolocation problem, Wang [8] proposed the Monte Carlo importance sampling (MCIS) technique. Also, the Gaussian distribution and probability density function (pdf) was used to obtain an efficient importance function.

In this paper, we obtain the optimal solution of geolocation problem using the IRLS scheme. The method of IRLS scheme is used to solve the optimization problem with scalar error denoted objective function by some iterative processes. With using the IRLS scheme, the weight matrix that decides the significance of each measurement data is derived

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with the scalar error of estimation. IRLS scheme based geolocation can obtain the optimal solution with limited TDOA data. The organization of our paper is as follows. The geolocation formula is derived with TDOA measurements in section II. In section III, the objective function that denotes the scalar error with L_p -norm approximation is formulated. Also, the optimal solution using the IRLS scheme based iteration process is expressed. Section IV shows the effectiveness of our proposed geolocation method through some simulations. Conclusion is presented finally in section V.

II. GEOLOCATION FORMULATION USING TDOA MEASUREMENTS

The geolocation equation using TDOA measurement data is derived in section II. Among various gelocation methods, TDOA based position estimation method is the most frequently used geolocation method due to its accuracy and versatility. If at least three base stations that can measure the signal of mobile are available, we can formulate the geolocation of a mobile in two dimensional coordinates. The unknown position of a mobile that we try to estimate is expressed as $\mathbf{m}=\begin{bmatrix} x & y \end{bmatrix}^T$. Let $\mathbf{b}_i=\begin{bmatrix} x_i & y_i \end{bmatrix}^T$, $i=1,\ 2,\cdots,\ N$ be a known position of base station. N is the number of base stations. For geolocation formulation, the TDOA measurement data can be obtained as following equations.

$$\begin{aligned} r_i &= v_s t_i , \\ r_{i1} &= r_i - r_1 , \\ &= \sqrt{\left(x-x_i\right)^2+\left(y-y_i\right)^2}-\sqrt{\left(x-x_1\right)^2+\left(y-y_1\right)^2}, \end{aligned} \tag{1}$$

where r_i is the distance from a mobile to base station that can be computed with signal propagation speed v_s and signal arrival time t_i at i-th base station. r_{i1} denotes the measured distance difference data between i-th base station and the first base station. The distance difference data can be computed with the time difference data of arrival signal without time synchronization at each base stations.

Under the assumption that the TDOA data do not contain measurement noise, the following linear equation can be derived with the ideal distance and distance difference variable r_i^0 and r_{i1}^0.

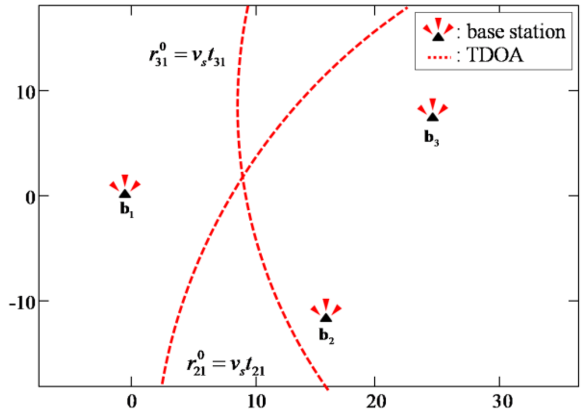

Fig. 1 Geolocation using TDOA measurements

$$\begin{aligned} &\left(x_i-x_1\right)\left(x-x_1\right)+\left(y_i-y_1\right)\left(y-y_1\right)+r_{i1}^0 r_1^0 \\ &=\frac{1}{2}\left(\left(x_i-x_1\right)^2+\left(y_i-y_1\right)^2-\left(r_{i1}^0\right)^2\right). \end{aligned} \tag{2}$$

In real environment, however, the measured TDOA data contains the measurement noise that disturbs the precise estimation of mobile position. The geolocation equation, denoted by Eq. (2), can be rewritten as a following matrix form with the noise contained measurement data.

$$\mathbf{G\rho}=\mathbf{d}, \tag{3}$$

with

$$\mathbf{G}=\begin{bmatrix} x_2-x_1 & y_2-y_1 & r_{21} \\ \vdots & \vdots & \vdots \\ x_N-x_1 & y_N-y_1 & r_{N1} \end{bmatrix},$$

$$\mathbf{d}=\begin{bmatrix} \frac{1}{2}\left(\left(x_2-x_1\right)^2+\left(y_2-y_1\right)^2-r_{21}^2\right) \\ \vdots \\ \frac{1}{2}\left(\left(x_N-x_1\right)^2+\left(y_N-y_1\right)^2-r_{N1}^2\right) \end{bmatrix}.$$

In equation (3), the variable vector $\boldsymbol{\rho}$ that is denoted by $\begin{bmatrix} x-x_1 & y-y_1 & r_1^0 \end{bmatrix}^T$ contains the estimated position of a mobile. However, the measurement noises in variables $\mathbf{G}$ and $\mathbf{d}$ cause the difference between the estimated position and the true position of a mobile. To obtain the optimized solution that minimizes the estimation error, we suggest the IRLS scheme based optimization solution in section III.

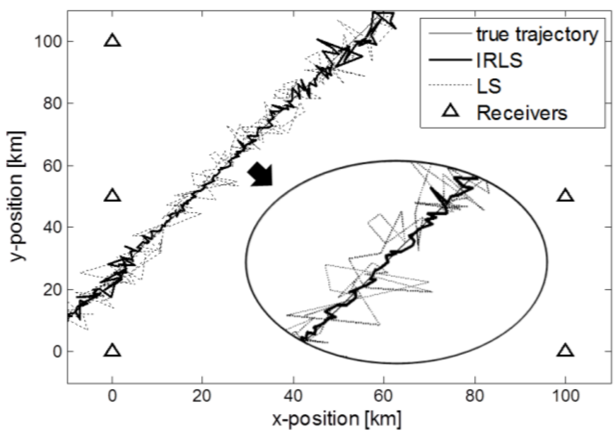

Fig. 2 Trajectory estimation using TDOA measurements

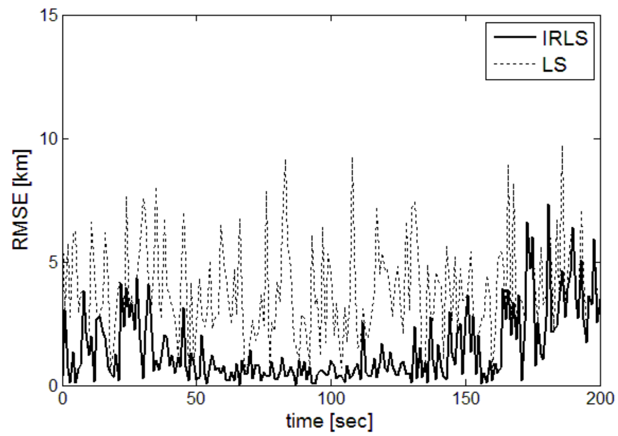

Fig. 3 RMSE comparison using IRLS and LS

III. OPTIMIZED SOLUTION USING IRLS SCHEME

In order to estimate the optimal solution that minimizes the estimation error, weighted least square (WLS) algorithm is applied to geolocation equation in general. For applying WLS algorithm to geolocation formula, it is significant to obtain the covariance variable that represents the weight of each measurement data. The method of IRLS scheme can solve the optimization problem with obtaining weight matrix using the L_p -norm approximation of the estimation error from previous iteration result. The geolocation objective function for minimizing estimation error can be formulated using Eq. (3) as follows

$$\arg\min_{\boldsymbol{\rho}} \sum_{i=1}^{N-1} \left| d_i - \boldsymbol{\Gamma}_i \boldsymbol{\rho} \right|^p \tag{4}$$

where

$$d_i = \frac{1}{2}\left(\left(x_i - x_1\right)^2 + \left(y_i - y_1\right)^2 - \left(r_{i1}^0\right)^2 \right)$$
$$\boldsymbol{\Gamma}_i = \begin{bmatrix} x_i - x_1 & y_i - y_1 & r_{i1}^0 \end{bmatrix}$$

In general, the weight matrix of WLS algorithm can be obtained with Euclidean norm (L_2 -norm). The definition of a scalar error of estimation using WLS algorithm can be derived as follows

$$\begin{aligned} \varepsilon_{WLS} &= \sum_{i}^{N-1} c_i^2 \left| d_i - \boldsymbol{\Gamma}_i \boldsymbol{\rho} \right|^2 \\ &= \mathbf{e}^T \mathbf{C}^T \mathbf{C} \mathbf{e} \end{aligned} \tag{5}$$

where the matrix variable $\mathbf{e}$ is the difference between left side and right side of Eq. (3). $\mathbf{C}$ means the weight matrix.

Using IRLS scheme, the WLS solution of j-th iteration can be obtained using the scalar error of (j-1)-th iteration result. The weight matrix of j-th iteration $\mathbf{C}(j)$ can be derived as following equation using Eq. (4) and Eq. (5).

$$\begin{aligned} &\mathbf{C}(j) = diag\left\{ c_1(j), \cdots, c_{N-1}(j) \right\} \\ &c_i(j) = \left| d_i - \boldsymbol{\Gamma}_i \boldsymbol{\rho}(j-1) \right|^{(p-2)/2} \end{aligned} \tag{6}$$

where $\boldsymbol{\rho}(j-1)$ is the IRLS solution of (j-1)-th iteration. The IRLS solution of j-th iteration which minimize the computed scalar error using L_p -norm can be obtained with reweighted covariance variable as follows

$$\boldsymbol{\rho}(j) = \left[\mathbf{G}^T \mathbf{C}^T(j) \mathbf{C}(j) \mathbf{G} \right]^{-1} \mathbf{G}^T \mathbf{C}^T(j) \mathbf{C}(j) \mathbf{d} \tag{7}$$

To derive the minimum norm approximate solution, we start the iteration with the weight matrix of the first iteration as the identity matrix $\mathbf{I}$. Precise solution which minimizes the estimation error can be obtained with the restricted measurement data through the IRLS scheme based iteration process using Eq. (7).

IV. SIMULATION RESULTS

In section IV, the effectiveness of our proposed IRLS scheme based geolocation using TDOA measurements is confirmed with some simulations. The active radio signal from a mobile is assumed to be measured at five base stations of which the positions are (0, 0), (0, 50), (0, 100), (100, 0), and (100, 50) km. In our simulations, the measurement noise in TDOA data follows a Gaussian distribution with a variance of 0.1. The simulated trajectory has 200 time samples with one second interval. For simplification of the computation, the signal propagation speed c is assumed as 1km/s.

Figure 2 describes the comparison of estimated trajectories using the general least square (LS) algorithm and the proposed IRLS algorithm. The thick solid line is the estimated trajectory using IRLS algorithm and the thin dotted means the estimated trajectory using LS algorithm. The thin solid line is the true trajectory of a mobile. In Fig. 2, the estimated trajectory using IRLS algorithm more closely follows the real trajectory than that of LS algorithm. The root mean square error (RMSE) comparison of IRLS and LS estimation is depicted in Fig. 3. The thick solid line and thin dotted line mean the RMSE using IRLS and LS at each time, respectively. As shown in Fig. 3, RMSE of a mobile position estimated by IRLS scheme is much less than RMSE of LS.

V. CONCLUSION

The geolocation method using TDOA measurement data was suggested in this paper. The measurement noise due to signal interference and disruption caused the estimation inaccuracy of a mobile position. In order to obtain the solution that minimizes the scalar error of estimation, the IRLS scheme based iteration process was introduced. The optimized weight matrix was computed using the error norm of the previous itertation. Although the measured TDOA data was limited, the optimal solution of geolocation problem was obtained through proposed IRLS scheme. The high performance of our proposed geolocation method was verified in some simulations.

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