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Linear Active Disturbance Rejection Control Using Plant Inverse Property

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Abstract: In this paper, we propose a control law in a linear active disturbance rejection control (LADRC) for removal of ramp disturbance. We use plant inverse characteristics as a control law. We calculate the steady-state error of the conventional method and that of the proposed method using the final-value theorem. In a conventional and a proposed LADRC, each plant output has no steady-state error when a step signal is assumed as a plant input-side disturbance. However, in a conventional LADRC, the plant output has a steady-state error when a ramp signal is assumed as a plant input-side disturbance. On the other hand, in a proposed method, the plant output has no steady-state error. In comparison with a conventional method, the proposed method has almost the same control performance for a plant with a modeling error.

Keywords: Active disturbance rejection control. Extended state observer. Steady-state error. Plant inverse characteristics.

I. INTRODUCTION

The origin of an active disturbance rejection control (ADRC) is a controller proposed by Han *et al* [1], [2]. By designing an extended state observer (ESO), unknown dynamics for plant model uncertainty and disturbances are estimated as a generalized disturbance. ADRC can compensate by estimating state variables and disturbance in real time and feeding back it [1], [3-5]. ADRC can control for not only a nonlinear mathematical model of a plant, but also a linear one. However, the structure of ADRC is complicated and there are many adjustment parameters. Therefore, linear version of ADRC (LADRC) has been proposed [4], [6], [7]. LADRC is composed of a linear ESO and a linear state feedback. Since LADRC has only two adjustment parameters, observer bandwidth ω_o and controller bandwidth ω_c, it is easy to tune.

Most of the papers about ADRC assume a step signal as a load disturbance. However, papers of ADRC concerning the ramp disturbance are few. Examples of a ramp disturbance are flow quantity change of the oil in the petrochemical plant and viscous frictional force when an equipment moves at uniform acceleration [8], [9]. It is essential to adapt such disturbances. In a conventional LADRC, the plant output has a steady-state error when a ramp disturbance is inserted.

To solve this problem, we propose a method to reduce an influence of the ramp disturbance by changing the control law. We use plant inverse property as a control law. The proposed method rejects not only step disturbance but also ramp disturbance. The steady-state error of the proposed method and that of the conventional method are calculated by the final-value theorem. In comparison with a conventional method, the proposed method has almost the same control performance for a plant with a modeling error.

II. TWO FORMULATIONS

This section describes the design method of ADRC.

A. Linear ADRC (LADRC)

In a linear time invariant (LTI) system, the plant transfer function $G_p(s)$ is

$$G_p(s) = \frac{Y(s)}{U(s)} = \frac{\sum_{j=0}^{m}(b_j + \Delta b_j)s^j}{s^n + \sum_{i=0}^{n-1}(a_j + \Delta a_j)s^i} \qquad (1)$$

where:
$U(s)$ is a Laplace transform of a plant input.
$Y(s)$ is a Laplace transform of a plant output.
$a_i (i = 0,1,\cdots,n-1)$ and $b_j (j = 0,1,\cdots,m)$ are nominal plant parameters.

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$\Delta a_i (i=0,1,\cdots,n-1)$ and $\Delta b_j (j=0,1,\cdots,m)$ are error parameters of a_i and b_j caused by the plant modeling error.

ADRC doesn't need the accurate model of the plant and the disturbance, but the relative order of $G_p(s)$, and its gain $b := b_m$. The inverse Laplace transform in (1) is calculated by

$$y^{(p)}(t) = f(t) + \tilde{u}(t) \quad (2)$$

$$\tilde{u}(t) = bu(t) \quad (3)$$

where:
$f(t)$ is a total disturbance including the unknown dynamics and the external disturbance.
$p = n - m$ means a relative degree of a plant.
$y(t)$ is a plant output.
$y^{(p)}(t)$ is an p-th derivative of $y(t)$.
$u(t)$ is a plant input.

State variables $x_i(t)\ (i=1,2,\cdots,n+1)$ are defined as

$$\begin{cases} x_1(t) = y(t) \\ x_2(t) = \dot{y}(t) \\ \quad\vdots \\ x_p(t) = y^{(p-1)}(t) \\ x_{p+1}(t) = f(t) \end{cases} \quad (4)$$

The time derivative of (4) is

$$\begin{cases} \dot{x}_1(t) = x_2(t) \\ \dot{x}_2(t) = x_3(t) \\ \quad\vdots \\ \dot{x}_p(t) = y^{(p)}(t) = x_{p+1}(t) + \tilde{u}(t) \\ \dot{x}_{p+1}(t) = \dot{f}(t) \end{cases} \quad (5)$$

The matrix form in (4) and (5) is

$$\begin{cases} \dot{\boldsymbol{x}}(t) = \boldsymbol{A_e}\boldsymbol{x}(t) + \boldsymbol{B_e}\tilde{u}(t) + \boldsymbol{E}\dot{f}(t) \\ y(t) = \boldsymbol{C_e}\boldsymbol{x}(t) \end{cases} \quad (6)$$

where:

$$\boldsymbol{x}(t) = [x_1(t) \quad x_2(t) \quad \cdots \quad x_p(t) \quad x_{p+1}(t)]^{\mathrm{T}}_{<p+1>\times 1}.$$

$$\boldsymbol{A_e} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix}_{<p+1>\times<p+1>}.$$

$$\boldsymbol{B_e} = [0 \quad 0 \quad \cdots \quad 1 \quad 0]^{\mathrm{T}}_{<p+1>\times 1}.$$

$$\boldsymbol{C_e} = [1 \quad 0 \quad \cdots \quad 0]_{1\times<p+1>}.$$

$$\boldsymbol{E} = [0 \quad 0 \quad \cdots \quad 0 \quad 1]^{\mathrm{T}}_{<p+1>\times 1}. \quad (7)$$

The p+1-th order ESO for (7) is

$$\begin{cases} \dot{\hat{\boldsymbol{x}}}(t) = \boldsymbol{A_e}\hat{\boldsymbol{x}}(t) + \boldsymbol{B_e}\tilde{u}(t) + \boldsymbol{L}\{y(t) - \hat{y}(t)\} \\ \hat{y}(t) = \boldsymbol{C_e}\hat{\boldsymbol{x}}(t) \end{cases} \quad (8)$$

where:
$\hat{\boldsymbol{x}}(t)$ is an estimated state vector.
$\boldsymbol{L}$ is an observer gain vector.
$\hat{y}(t)$ is an estimated plant output.

Here, $\boldsymbol{L}$ can generally be determined by a pole placement method, so that the eigenvalues of $\boldsymbol{A_e} - \boldsymbol{LC_e}$ are $-\omega_o$.

The block diagram of a linear ADRC is shown in Fig. 1.

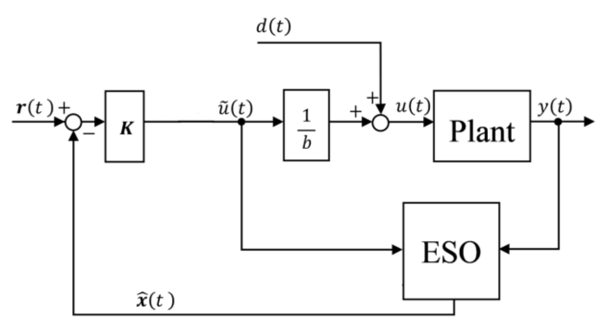

Fig. 1 Structure of LADRC

The control law is represented by

$$u(t) = \frac{\boldsymbol{K}\{\boldsymbol{r}(t) - \hat{\boldsymbol{x}}(t)\}}{b} \quad (9)$$

where:
$\boldsymbol{K} = [k_1\ k_2 \cdots k_p\ 1]$.

$\boldsymbol{r}(t)=[r(t)\;\dot{r}(t)\cdots r^{(p-1)}(t)\;r^{(p)}]^{\mathrm{T}}$.
$k_i={}_p\mathrm{C}_i\omega_c^{p+1-i}(i=1,2,\cdots,p)$.
${}_p\mathrm{C}_i$ is a binomial coefficient.

B. Modified ADRC (MADRC)

If we use an inverse model for a plant $G_{inv}(s)$ and p integrators substituting for $1/b$, modified control law is as described below.

$$U(s)=G_{inv}(s)\cdot\frac{1}{s^p}\cdot\tilde{U}(s) \tag{10}$$

where:
$1/s^p$ describes p integrators to satisfy a proper transfer function.

$G_{inv}(s)$ is given as

$$G_{inv}(s)=\frac{s^n+\sum_{i=0}^{n-1}a_i s^i}{\sum_{j=0}^{m}b_j s^j} \tag{11}$$

The block diagram of the modified ADRC is shown in Fig. 2.

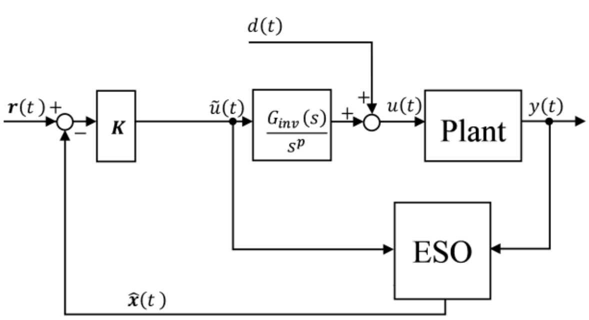

Fig. 2 Structure of MADRC

From Fig. 1 and Fig. 2, the Laplace transform of an input $u(t)$ is

$$U(s)=\frac{\boldsymbol{K}\left\{\boldsymbol{R}(s)-\hat{\boldsymbol{X}}(s)\right\}}{C(s)}+D(s) \tag{12}$$

where:
$\boldsymbol{R}(s)=[1\;s\cdots s^{p-1}\;0]^{\mathrm{T}}R(s)$.

$R(s)$ is the Laplace transform of the reference signal $r(t)$.
$D(s)$ is the Laplace transform of an input-side disturbance $d(t)$.

If you use the LADRC, $C(s)=b$. If you use the MADRC, $C(s)=s^p/G_{inv}(s)$.

Arranging (8) and solving it for $\hat{\boldsymbol{X}}(s)$, we get

$$\begin{aligned}\hat{\boldsymbol{X}}(s)=&\{s\boldsymbol{I}_{p+1}-\boldsymbol{A_e}+\boldsymbol{B_e K}+\boldsymbol{LC}(s\boldsymbol{I}_n-\boldsymbol{A})^{-1}\boldsymbol{BK}/C(s)\\&+\boldsymbol{LC_e}\}^{-1}[\{\boldsymbol{B_e K}+\boldsymbol{LC}(s\boldsymbol{I}_n-\boldsymbol{A})^{-1}\boldsymbol{BK}/C(s)\}\boldsymbol{R}(s)\\&+\boldsymbol{LC}(s\boldsymbol{I}_n-\boldsymbol{A})^{-1}\boldsymbol{B}D(s)],\end{aligned} \tag{13}$$

where:
$\boldsymbol{I}_n$ is an identity matrix of the n dimension.

From Fig. 1 and Fig. 2, $Y(s)$ can be described as

$$Y(s)=\boldsymbol{C}(s\boldsymbol{I}_n-\boldsymbol{A})^{-1}\boldsymbol{B}\left[\boldsymbol{K}\left\{\boldsymbol{R}(s)-\hat{\boldsymbol{X}}(s)\right\}/C(s)+D(s)\right]. \tag{14}$$

By substituting (13) to (14), $Y(s)$ can be described as

$$Y(s)=P(s)R(s)+P_d(s)D(s) \tag{15}$$

where:

$$\begin{aligned}P(s)=&\boldsymbol{C}(s\boldsymbol{I}_n-\boldsymbol{A})^{-1}\boldsymbol{BK}/C(s)-\boldsymbol{C}(s\boldsymbol{I}_n-\boldsymbol{A})^{-1}\boldsymbol{BK}\\&\{s\boldsymbol{I}_{p+1}-\boldsymbol{A_e}+\boldsymbol{B_e K}+\boldsymbol{LC}(s\boldsymbol{I}_n-\boldsymbol{A})^{-1}\boldsymbol{BK}/C(s)\\&+\boldsymbol{LC_e}\}^{-1}\{\boldsymbol{B_e K}+\boldsymbol{LC}(s\boldsymbol{I}_n-\boldsymbol{A})^{-1}\boldsymbol{BK}/C(s)\}.\end{aligned} \tag{16}$$

$$\begin{aligned}P_d(s)=&\boldsymbol{C}(s\boldsymbol{I}_n-\boldsymbol{A})^{-1}\boldsymbol{B}-\boldsymbol{C}(s\boldsymbol{I}_n-\boldsymbol{A})^{-1}\boldsymbol{BK}\\&(s\boldsymbol{I}_{p+1}-\boldsymbol{A_e}+\boldsymbol{B_e K}+\boldsymbol{LC}(s\boldsymbol{I}_n-\boldsymbol{A})^{-1}\boldsymbol{BK}/C(s)\\&+\boldsymbol{LC_e})^{-1}\boldsymbol{LC}(s\boldsymbol{I}_n-\boldsymbol{A})^{-1}\boldsymbol{B}.\end{aligned} \tag{17}$$

By using the final-value theorem, we calculate the steady-state error e_{ss} as shown below

$$e_{ss}=\lim_{t\to\infty}e(t)=\lim_{s\to 0}sE(s) \tag{18}$$

where:

$e(t) = r(t) - y(t)$.

The Laplace transform of $e(t)$ is described as

$E(s) = R(s) - Y(s) = (1 - P(s))R(s) - P_d(s)D(s)$.

$R(s)$ is the Laplace transform $1/s$ of a unit step input

$D(s)$ is the Laplace transform k_s/s of a step disturbance or k_r/s^2 of a ramp disturbance.

k_s is a magnitude of the step disturbance.

k_r is a magnitude of the ramp disturbance. Then, e_{ss} is summarized as shown in table.1.

Table.1 Steady-state error e_{ss}

Relative order	LADRC	MADRC
1	$\frac{k_r c(\omega_c + 2\omega_o)}{\omega_c \omega_o^2}$	0
2	$\frac{k_r c(\omega_c^2 + 6\omega_c\omega_o + 3\omega_o^2)}{\omega_c^2 \omega_o^3}$	0
3	$\frac{k_r c(\omega_c^3 + 12\omega_c^2\omega_o + 18\omega_c\omega_o^2 + 4\omega_o^3)}{\omega_c^3 \omega_o^4}$	0

III. SIMULATIONS

A. Simulations Setups

We performed simulations for five kinds of plants. When the plant is a nominal model, $(\Delta b_2, \Delta b_1, \Delta b_0, \Delta a_2, \Delta a_1, \Delta a_0) = (0,0,0,0,0,0)$. If the plant has a modeling error, we assume that $(\Delta b_2, \Delta b_1, \Delta b_0, \Delta a_2, \Delta a_1, \Delta a_0) = (0.5b_2, 0.5b_1, 0.5b_0, 0.5a_2, 0.5a_1, 0.5a_0)$. In simulation, $(\omega_o, \omega_c, k_s, k_r) = (50,10,1,1)$. A unit step signal and a unit ramp signal are assumed as a plant input-side disturbance. One of these disturbances is loaded at eight seconds.

plant1: $G_p(s) = \frac{50s + 200}{s^2 + 11s + 10}$

plant2: $G_p(s) = \frac{175s^2 + 525s + 350}{s^3 + 10s^2 + 29s + 20}$

plant3: $G_p(s) = \frac{340}{s^2 + 11s + 10}$

plant4: $G_p(s) = \frac{50s + 200}{s^3 + 10s^2 + 29s + 20}$

plant5: $G_p(s) = \frac{200}{s^3 + 10s^2 + 29s + 20}$

B. Simulation Results

From Figs. 3 to 12 show simulation results of output responses. As shown in these results, both methods have superior robustness for the plant with a modeling error. From Figs. 3, 5, 7, 9 and 11, in LADRC and MADRC, each plant output has no steady-state error when a step signal is assumed as a plant input-side disturbance. From Figs. 4, 6, 8, 10 and 12, in LADRC, the plant output has a steady-state error when a ramp signal is assumed as a plant input-side disturbance. On the other hand, in MADRC, the plant output has no steady-state error.

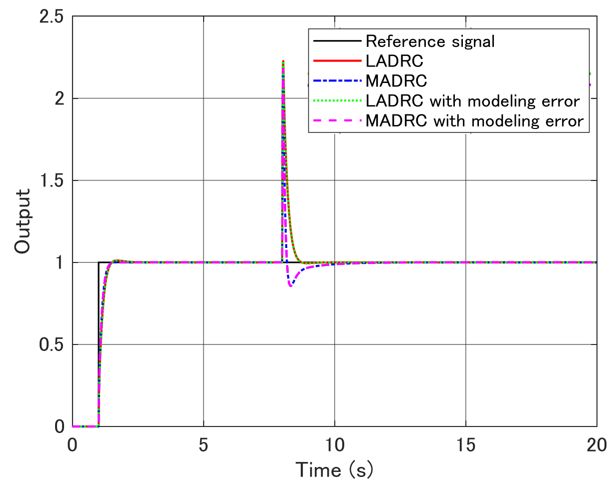

Fig. 3 Output of plant1 (step disturbance)

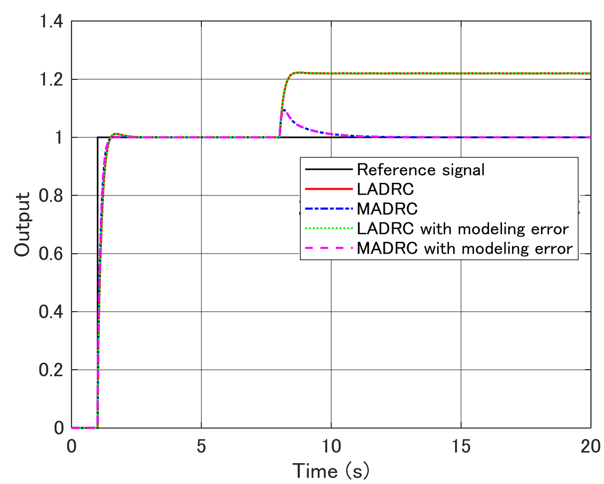

Fig. 4 Output of plant1 (ramp disturbance)

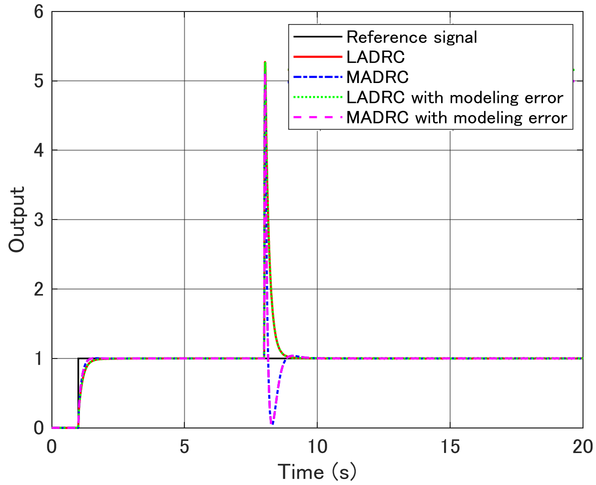

Fig. 5 Output of plant2 (step disturbance)

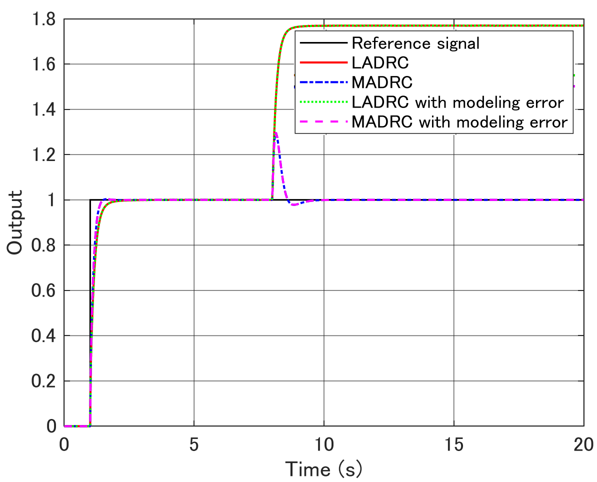

Fig. 6 Output of plant2 (ramp disturbance)

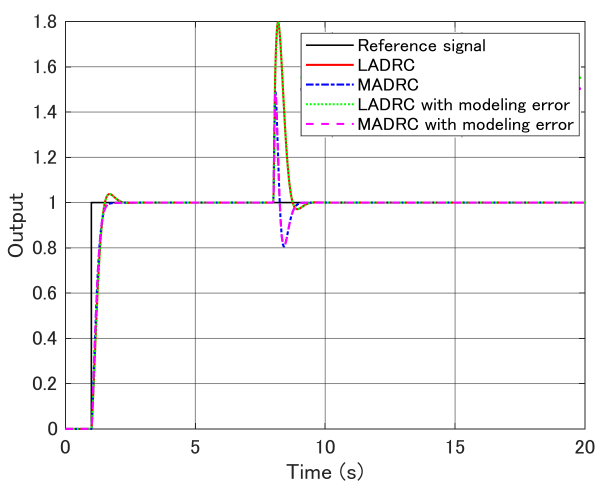

Fig. 7 Output of plant3 (step disturbance)

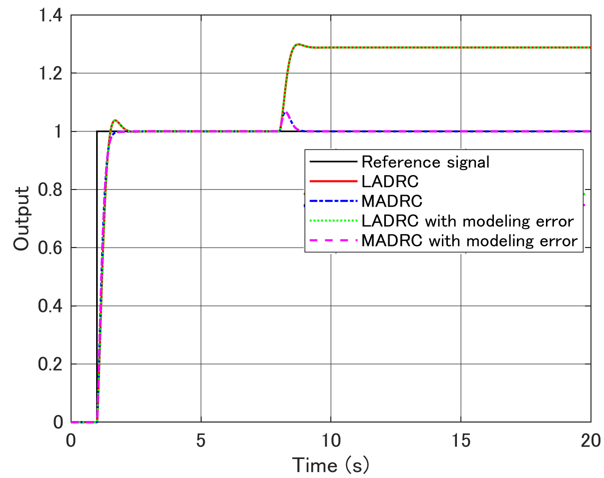

Fig. 8 Output of plant3 (ramp disturbance)

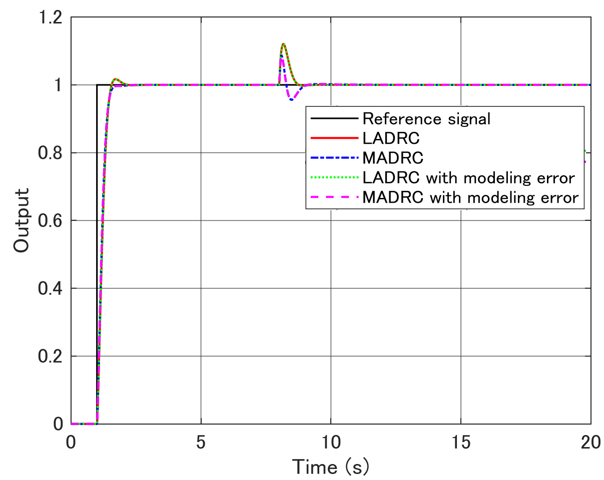

Fig. 9 Output of plant4 (step disturbance)

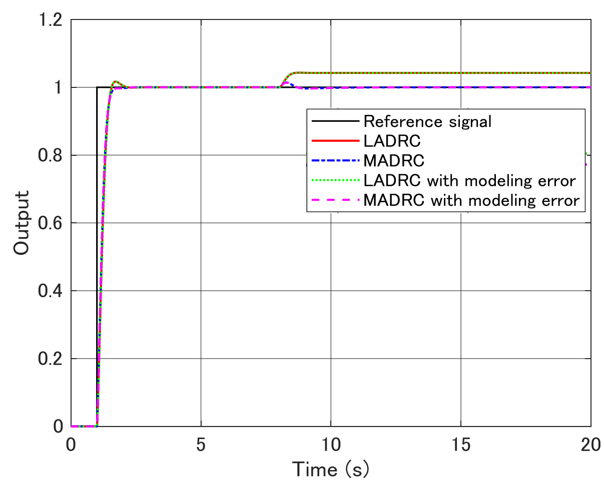

Fig. 10 Output of plant4 (ramp disturbance)

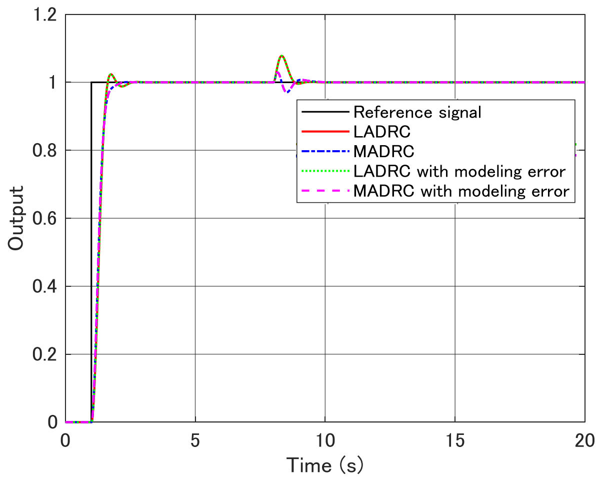

Fig. 11 Output of plant5 (step disturbance)

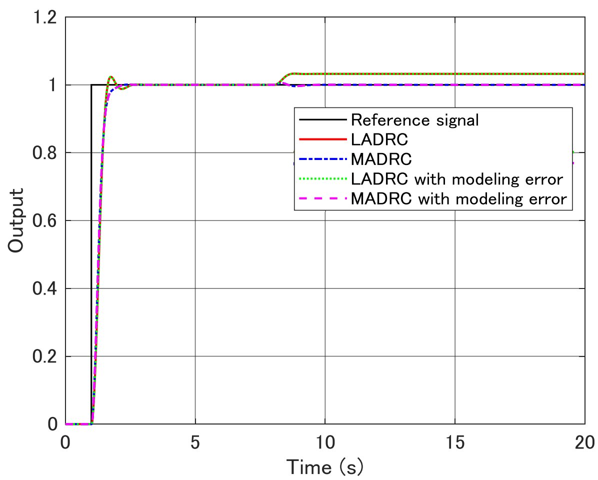

Fig. 12 Output of plant5 (ramp disturbance)

IV. CONCLUSIONS

We have proposed a modified control law in ADRC. We also have proved that steady-state error is zero using the final-value theorem. In simulations, we have confirmed that the proposed method has an excellent robustness for a plant with a modeling error.

REFERENCES

[1] J. Han. From PID to active disturbance rejection control. *IEEE Trans. Ind. Electron.*, vol. 56, no. 3, pp: 900-906, 2009.

[2] Z. Gao, Y. Huang and J. Han. An alternative paradigm for control system design. *Proceedings of the 40th IEEE Conf. decision and control.* pp:4578-4585, Dec., 2001, Florida.

[3] J. Tatsumi and Z. Gao. On the enhanced ADRC design with a low observer bandwidth. *Proceedings of the 32nd Chinese Control Conference*, pp: 297-302, Jul. 26-28, 2013, Xi'an.

[4] W. Tan and C. Fu. Linear active disturbance-rejection control: Analysis and tuning via IMC. *IEEE Trans. Ind. Electron.*, vol. 63, no. 4, pp: 2350-2359, 2016.

[5] R. Tanaka, T. Koga, and K. Ikeda. A study of determining an adaptive control input without using a parallel feedforward compensator. *Proceedings of the 9th International Conference of Information Technology and Electrical Engineering*, pp: 158-163, Oct. 12-13, 2017, Phuket.

[6] C. Fu and W. Tan. A new method to tune linear active disturbance rejection controller. *Proceedings of the American Control Conference*, pp:1560-1565, Jul. 6-8, 2011, Boston.

[7] Z. Gao. Scaling and bandwidth-parameterization based controller tuning. *Proceedings of the American control conference.* pp:4989-4996, Jun., 2003, Colorado.

[8] T. Satoh, K. Kaneko, and N. Saito. Improvement of tracking performance in predictive functional control using disturbance observer. *Journal of the Japan Society of Mechanical Engineers*, vol. 10, no. 15, pp: 132-139, 2011 (in Japanese).

[9] T. Satoh, H. Hara, J. Nagase, N. Saito, and N. Saga. Design of an estimator-based predictive functional control for ramp disturbance suppression. *Journal of the Japan Society of Mechanical Engineers*, vol. 79, no. 808, pp: 365-375, 2013 (in Japanese).